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On the Existence of Correlated Equilibria

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On the Existence of Correlated Equilibria

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Abstract: We provide sufficient conditions which guarantee the existence of correlated equilibria in noncooperative games with finitely many players.

1. INTRODUCTION

A basic solution concept for noncooperative games which has found several applications in economics is that of a Nash equilibrium [Nash (1951)]. Recently, Aumann (1974, 1987) introduced the idea of a correlated equilibrium as an extension of the Nash equilibrium in order to allow for correlation between the player's randomizations. This notion can be described as follows: Let Ω denote the set of states of nature of the world and consider the game $\Gamma = \{(X_i, u_i, S_i, \mu_i) :$

$i=1,2,\dots,m\}$ where, X_i is the strategy set of player i , $u_i : \prod_{j=1}^m X_j \rightarrow \mathbb{R}$

is the payoff of player i , S_i is a partition of Ω denoting the private information of player i and μ_i is the prior of player i —a probability measure on Ω . For simplicity we assume that players have the same prior, i.e., $\mu_i \equiv \mu$ for all i . A strategy of player i is an S_i -measurable function $x_i : \Omega \rightarrow X_i$. An m -tuple of strategies $(x_1^*, x_2^*, \dots, x_m^*)$ is said to be a correlated equilibrium for the game Γ , if for all i , ($i=1,2,\dots,m$)

$$\sum_{\omega \in \Omega} u_i(x_1^*(\omega), x_2^*(\omega), \dots, x_m^*(\omega)) \mu(\omega) \geq \sum_{\omega \in \Omega} u_i(x_1^*(\omega), \dots, x_i, \dots, x_m^*(\omega)) \mu(\omega)$$

for every strategy x_i .¹

A fundamental question which arises is under what conditions the game Γ has a correlated equilibrium. The purpose of this paper is to provide an answer to this question. In particular, we show that if Ω is a probability measure space, X_i is a (Banach) set-valued function defined on Ω whose values are convex and weakly compact and u_i is

weakly continuous, integrably bounded and quasi-concave in the i^{th} coordinate, then the game Γ has a correlated equilibrium.² Moreover, adopting an idea of Schmeidler (1976), i.e., by viewing the extreme points of the set X_i as pure strategies we also prove a pure strategy correlated equilibrium existence result for the game Γ .³

The proofs of our results require arguments of a rather novel-type. However, it should be noted that the techniques which have been used in the literature of the existence of a Nash equilibrium in games with a continuum of players [e.g., Schmeidler (1976), Khan (1986), Yannelis (1987) and Yannelis-Rustichini (1988)] will turn out to be useful. Moreover, some theorems of Aumann (1965) on the integration of set-valued functions as well as extensions of Aumann's results given by Datko (1973), Khan (1986), Rustichini (1987) and Yannelis (1988) will play an important role.

The paper is organized in the following way: Section 2 contains notation and definitions. The main results of the paper are stated in Section 3 and their proof are collected in Sections 4 and 5. Finally, some concluding remarks are given in Section 6.

2. NOTATION AND DEFINITIONS

2.1 Notation

2^A denotes the set of all nonempty subsets of the set A ,

$\text{con } A$ denotes the convex hull of the set A ,

$\overline{\text{con } A}$ denotes the closed convex hull of the set A ,

R^ℓ denotes the ℓ -fold Cartesian product of the set of real numbers R ,

\emptyset denotes the empty set.

If X is linear topological space, its dual is the space X^* of all continuous linear functionals on X , and if $p \in X^*$ and $x \in X$ the value of p at x is denoted by $\langle p, x \rangle$.

2.2 Definitions

Let X, Y be topological spaces. The correspondence $\phi : X \rightarrow 2^Y$ is said to be upper semicontinuous (u.s.c.) if the set $\{x \in X : \phi(x) \subset V\}$ is open in X for every open subset V of Y . The graph of the correspondence $\phi : X \rightarrow 2^Y$ is denoted by $G_\phi = \{(x, y) \in X \times Y : y \in \phi(x)\}$.

Let $(\Omega, \mathcal{F}, \mu)$ be a complete finite measure space and X be a separable Banach space. The correspondence $\phi : \Omega \rightarrow 2^X$ is said to have a measurable graph if $G_\phi \in \mathcal{F} \otimes \mathcal{B}(X)$, where $\mathcal{B}(X)$ denotes the Borel σ -algebra on X and \otimes denotes product σ -algebra. The correspondence $\phi : \Omega \rightarrow 2^X$ is said to be lower measurable if for every open subset V of X the set $\{\omega \in \Omega : \phi(\omega) \cap V \neq \emptyset\}$ is an element of \mathcal{F} . It is well known that if $\phi : \Omega \rightarrow 2^X$ has a measurable graph it is also lower measurable [Himmelberg (1975, p. 47)]. Furthermore, if $\phi(\cdot)$ is closed valued and lower measurable then $\phi(\cdot)$ has a measurable graph.

Below we define the notion of a Bochner integrable function. We will follow the treatment of Diestel-Uhl (1977). Let $(\Omega, \mathcal{F}, \mu)$ be a finite measure space and X be a Banach space. A function $f : \Omega \rightarrow X$ is called simple if there exist x_1, x_2, \dots, x_n in X and $\sigma_1, \sigma_2, \dots, \sigma_n$ in \mathcal{F}

such that $f = \sum_{i=1}^n x_i \chi_{\sigma_i}$, where $\chi_{\sigma_i}(\omega) = 1$ if $\omega \in \sigma_i$ and $\chi_{\sigma_i}(\omega) = 0$ if

$\omega \notin \sigma_i$. A function $f : \Omega \rightarrow X$ is said to be μ -measurable if there exists a sequence of simple functions $f_n : \Omega \rightarrow X$ such that

$\lim_{n \rightarrow \infty} \|f_n(\omega) - f(\omega)\| = 0$ μ -a.e. A μ -measurable function $f : \Omega \rightarrow X$ is

said to be Bochner integrable if there exists a sequence of simple functions $\{f_n : n=1,2,\dots\}$ such that

$$\lim_{n \rightarrow \infty} \int_{\Omega} \|f_n(\omega) - f(\omega)\| d\mu(\omega) = 0.$$

In this case we define for each $A \in \mathcal{F}$ the integral to be $\int_A f(\omega) d\mu(\omega) = \lim_{n \rightarrow \infty} \int_A f_n(\omega) d\mu(\omega)$. It is a standard result [Diestel-Uhl (1977, Theorem

2, p. 45)] that if $f : \Omega \rightarrow X$ is a μ -measurable function then f is

Bochner integrable if and only if $\int_{\Omega} \|f(\omega)\| d\mu(\omega) < \infty$. We denote by

$L_1(\mu, X)$ the space of equivalence classes of X -valued Bochner inte-

grable functions $x : \Omega \rightarrow X$ normed by $\|x\| = \int_{\Omega} \|x(\omega)\| d\mu(\omega)$. It is well

known that normed by the functional $\|\cdot\|$ above, $L_1(\mu, X)$ becomes a

Banach space [Diestel-Uhl (1977, p. 50)]. We denote by S_{ϕ} the set of

all X -valued Bochner integrable selections from the correspondence

$\phi : \Omega \rightarrow 2^X$, i.e.,

$$S_{\phi} = \{y \in L_1(\mu, X) : y(\omega) \in \phi(\omega) \text{ } \mu\text{-a.e.}\}.$$

Following Aumann (1965) the integral of the correspondence $\phi : \Omega \rightarrow 2^X$ is defined as follows:

$$\int_{\Omega} \phi(\omega) d\mu(\omega) = \left\{ \int_{\Omega} y(\omega) d\mu(\omega) : y \in S_{\phi} \right\}.$$

We will denote the above integral by $\int \phi$. A correspondence $\phi : \Omega \rightarrow 2^X$

is said to be integrably bounded if there exists a map $g \in L_1(\mu, \mathbb{R})$

such that $\sup\{\|x\| : x \in \phi(\omega)\} \leq g(\omega)$ μ -a.e. Note that if Ω is a com-

plete finite measure space, and X is a separable Banach space and

$\phi : \Omega \rightarrow 2^X$ is a nonempty-valued correspondence with a measurable graph (or equivalently $\phi(\cdot)$ is lower measurable and closed valued), then $\phi : \Omega \rightarrow 2^X$ admits a measurable selection, i.e., there exists a measurable function $f : \Omega \rightarrow X$ such that $f(\omega) \in \phi(\omega)$ μ -a.e. By virtue of this result and provided that the correspondence $\phi : \Omega \rightarrow 2^X$ is integrably bounded, we can conclude that S_ϕ is nonempty and hence $\int \phi$ is nonempty as well.

A Banach space X has the Radon-Nikodym Property with respect to the measure space $(\Omega, \mathcal{J}, \mu)$ if for each μ -continuous vector measure $G : \mathcal{J} \rightarrow X$ of bounded variation there exists $g \in L_1(\mu, X)$ such that $G(E) = \int_E g(\omega) d\mu(\omega)$ for all $E \in \mathcal{J}$. A Banach space X has the Radon-Nikodym Property (RNP) if X has the RNP with respect to every finite measure space. It is a standard result [Diestel-Uhl (1977, Theorem 1, p. 98)] that if $(\Omega, \mathcal{J}, \mu)$ is a finite measure space $1 \leq p < \infty$, and X is a Banach space, then X^* has the RNP if and only if $(L_p(\mu, X))^* = L_q(\mu, X^*)$ where $\frac{1}{p} + \frac{1}{q} = 1$.

We conclude this section by noting that if X is a separable Banach space, $(\Omega, \mathcal{J}, \mu)$ is a finite positive measure space and $\phi : \Omega \rightarrow 2^X$ is an integrably bounded, nonempty, weakly compact and convex valued correspondence then by Diestel's Theorem [see Diestel (1977) or Papageorgiou (1985)] we have that S_ϕ is weakly compact in $L_1(\mu, X)$.

3. THE MAIN THEOREMS

Let Y be a separable Banach space whose dual Y^* has the RNP and $(\Omega, \mathcal{J}, \mu)$ be a separable, complete, finite, probability measure space. As usual we interpret Ω as the states of nature of the world and

assume that Ω includes all the events that we consider to be interesting, \mathcal{J} will denote the σ -algebra of events and obviously μ is the measure on (Ω, \mathcal{J}) .

A game $\Gamma = \{(X_i, u_i, S_i) : i=1, 2, \dots, m\}$ is a set of triplets (X_i, u_i, S_i) where,

- (1) $X_i : \Omega \rightarrow 2^Y$ is the strategy correspondence of player i ,
- (2) $u_i : \prod_{j=1}^m Y_j \rightarrow R$ is the payoff function of player i , and
- (3) S_i is the private information of player i , where S_i is a partition of (Ω, \mathcal{J}) . (Note that the probability measure μ on (Ω, \mathcal{J}) can be interpreted as the common prior of each player.)

Denote by L_{X_i} the set of all Bochner integrable and S_i -measurable selections from the strategy set X_i of player i , i.e., $L_{X_i} = \{x_i \in L_1(\mu, Y) : x_i : \Omega \rightarrow Y \text{ is } S_i\text{-measurable and } x_i(\omega) \in X_i(\omega) \mu\text{-a.e.}\}$.

Let $L_X = \prod_{i=1}^m L_{X_i}$. Moreover, let $L_{\tilde{X}_i} = \prod_{j \neq i} L_{X_j}$ and denote the points of $L_{\tilde{X}_i}$ by \tilde{x}_i .

The expected payoff of player i is a function $g_i : L_X \rightarrow R$ defined by

$$g_i(x) = \int_{\omega \in \Omega} u_i(x(\omega)) d\mu(\omega).$$

A strategy $x^* \in L_X$ is said to be a correlated equilibrium for the game Γ , if for all i , ($i=1, 2, \dots, m$)

$$g_i(x^*) = \max_{x_i \in L_{X_i}} g_i(x_i, \tilde{x}_i^*).$$

We are now ready to state the following result.

Theorem 3.1: Let $\Gamma = \{(X_i, u_i, S_i) : i=1,2,\dots,m\}$ be a game satisfying for all i the following assumptions:

(a.3.0) The partition S_i of (Ω, \mathcal{Y}) is countable,

(a.3.1) $X_i : \Omega \rightarrow 2^Y$ is an integrably bounded, weakly compact, convex, nonempty valued and S_i -lower measurable correspondence,⁵

(a.3.2) u_i is weakly continuous and integrably bounded,

(a.3.3) u_i is quasi-concave in the i^{th} coordinate, i.e., for each $\tilde{x}_i \in \tilde{X}_i = \prod_{j \neq i} X_j$, $u_i(x_i, \tilde{x}_i)$ is a quasi-concave function of x_i on X_i .

Then Γ has a correlated equilibrium.

We now turn to the existence of a pure strategy correlated equilibrium. From now on until the end of this section we set $Y = \mathbb{R}^{\ell}$. Hence, for the game Γ described above X_i is a correspondence from Ω into $2^{\mathbb{R}^{\ell}}$. Moreover, we assume that for each i , S_i is a sub σ -algebra of \mathcal{Y} and the restriction of μ to S_i is still denoted by μ . We denote by X_i^e the extreme points of X_i .

A pure strategy correlated equilibrium for the game Γ is an $x^* : \Omega \rightarrow \prod_{i=1}^m X_i^e$ such that each $x_i^*(\cdot)$ is S_i -measurable and for all i , $(i=1,2,\dots,m)$

$$\int_{\omega \in \Omega} u_i(x^*(\omega)) d\mu(\omega) \geq \int_{\omega \in \Omega} u_i(x_i(\omega), \tilde{x}_i^*(\omega)) d\mu(\omega)$$

for any S_i -measurable function $x_i : \Omega \rightarrow X_i^e$.

We are now ready to state our second main result.

Theorem 3.2: Let $\Gamma = \{(X_i, u_i, S_i) : i=1,2,\dots,m\}$ be a game satisfying for each i the following assumptions:

(a.3.4) (Ω, S_i, μ) is a separable, complete, finite, atomless probability measure space,

(a.3.5) $X_i : \Omega \rightarrow 2^{R^L}$ is an integrably bounded, compact, convex, nonempty valued and S_i -lower measurable correspondence,

(a.3.6) u_i is continuous, integrably bounded and linear on $\prod_{j=1}^m Y_j$.

Then Γ has a pure strategy correlated equilibrium.

The next two sections are devoted to the proofs of our theorems.

4. PROOF OF THEOREM 3.1

Let $L_{X_i} = \prod_{j \neq i} L_{X_j}$. For each i define the correspondence

$\phi_i : L_{X_i} \rightarrow 2^{L_{X_i}}$ by

$$\phi_i(\tilde{x}_i) = \{y_i \in L_{X_i} : g_i(y_i, \tilde{x}_i) = \max_{x_i \in L_{X_i}} g_i(x_i, \tilde{x}_i)\}.$$

Also define the correspondence $\phi : L_X \rightarrow 2^{L_X}$ by

$$\phi(x) = \prod_{i=1}^m \phi_i(\tilde{x}_i).$$

We must show that the set-valued function $\phi : L_X \rightarrow 2^{L_X}$ satisfies all the conditions of the Fan-Glicksberg fixed point theorem [Glicksberg (1952), see also Berge (1963, p. 251)]. Clearly, a fixed point of ϕ

is a correlated equilibrium for the game Γ . We begin by proving the following fact:

Fact 4.1: L_X is weakly compact, nonempty, convex and metrizable.

Proof: It follows directly from Diestel's theorem that L_{X_i} is weakly compact and therefore L_X is weakly compact as well. Obviously, since each $X_i(\cdot)$ is convex valued, L_{X_i} is convex and so is L_X . Since $X_i(\cdot)$ is closed valued and S_i -lower measurable (assumption (a.3.1)) it follows from the Kuratowski and Ryll-Nardzewski measurable selection theorem [Himmelberg (1975, p. 60)] that there exists an S_i -measurable function $f_i : \Omega \rightarrow Y$ such that $f_i(\omega) \in X_i(\omega)$ μ -a.e. Since X_i is integrably bounded f_i is also Bochner integrable, i.e., $f_i \in L_1(\mu, Y)$ and therefore we can conclude that $f_i \in L_{X_i}$. Consequently, L_{X_i} is nonempty and so is $L_X = \prod_{i=1}^m L_{X_i}$. Note that since $(\Omega, \mathcal{F}, \mu)$ is a

separable measure space and Y is a separable Banach space, $L_1(\mu, Y)$ is separable. Since L_{X_i} is a weakly compact subset of the separable Banach space $L_1(\mu, Y)$ it is also metrizable [Dunford-Schwartz (1958, Theorem V.6.3, p. 434)]. Finally, since the set of players is finite, L_X is metrizable [as it is the finite product of metrizable spaces, Berge (1963, Theorem 2, p. 86)]. This completes the proof of the claim.

Fact 4.2: For each i , ($i=1,2,\dots,m$), $g_i(\cdot)$ is weakly continuous.

Proof: Let $\{x_n : n=1,2,\dots\}$ be a sequence in $L_X = \prod_{i=1}^m L_{X_i}$ converging weakly⁶ to $x \in L_X$, i.e., for each i , ($i=1,2,\dots,m$) the sequence

$\{x_n^i : n=1,2,\dots\}$ in L_{X_i} converges weakly to x^i in L_{X_i} . If we show that for all i , x_n^i converges pointwise in the weak topology of X_i to x^i , then given the fact that u_i is integrably bounded and weakly continuous the result will follow from the Lebesgue dominated convergence theorem. Note that since for each i the partition S_i is countable, i.e., $S_i = \{\alpha_i^1, \alpha_i^2, \alpha_i^3, \dots\}$ the fact that x_n^i and x^i are elements of L_{X_i} implies that $x_n^i = \sum_{k=1}^{\infty} x_n^{i,k} \chi_{\alpha_i^k}$, $x^i = \sum_{k=1}^{\infty} x^{i,k} \chi_{\alpha_i^k}$ for $x_n^{i,k}$, $x^{i,k}$ in X_i and therefore we can conclude that x_n^i converges pointwise in the weak topology of X_i to x^i . This completes the proof of the claim.

It follows from Berge's maximum theorem [Berge (1963, p. 116)] that for each i , $\phi_i(\cdot)$ is weakly u.s.c. Moreover, since for each i , u_i is quasi-concave in the i^{th} coordinate it follows that for each i and each $\tilde{x}_i \in L_{X_i}$, $g_i(x_i, \tilde{x}_i)$ is a quasi-concave function of x_i on L_{X_i} and consequently for each i , $\phi_i(\cdot)$ is convex valued. Since $g_i(\cdot)$ is weakly continuous and L_{X_i} is weakly compact, $\phi_i(\cdot)$ is nonempty valued. Thus, $\phi_i : L_{X_i} \rightarrow 2^{L_{X_i}}$ is a weakly u.s.c., closed, convex, nonempty valued correspondence and so is $\Phi : L_X \rightarrow 2^{L_X}$ defined by $\Phi(x) = \prod_{i=1}^m \phi_i(\tilde{x})$, [Berge (1963, Theorem 4', p. 114)]. Consequently, the set-valued function $\Phi : L_X \rightarrow 2^{L_X}$ satisfies all the conditions of the Fan-Glicksberg fixed point theorem and therefore there exists $x^* \in L_X$ such that $x^* \in \Phi(x^*)$. It can be easily checked that the fixed point is by construction a correlated equilibrium for the game Γ . This completes the proof of Theorem 3.1.

5. PROOF OF THEOREM 3.2

Denote by $L_{X_i}^e$ the set of all Bochner integrable and S_i -measurable selections from the strategy set X_i^e of player i , i.e.,

$$L_{X_i}^e = \{x_i \in L_1(\mu, \mathbb{R}^l) : x_i : \Omega \rightarrow \mathbb{R}^l \text{ is } S_i\text{-measurable and}$$

$$x_i(\omega) \in X_i^e(\omega) \text{ } \mu\text{-a.e.}\}. \text{ Let } L_X^e = \prod_{i=1}^m L_{X_i}^e. \text{ Define the mapping}$$

$$\pi : L_1(\mu, \mathbb{R}^l) \rightarrow \mathbb{R}^l \text{ by } \pi(y) = \int_{\omega \in \Omega} y(\omega) d\mu(\omega). \text{ The integral of the set-}$$

$$\text{valued function } X_i^e \text{ is } \int_{\omega \in \Omega} X_i^e(\omega) d\mu(\omega) = \pi(L_{X_i}^e) = \{\tau(y) : y \in L_{X_i}^e\}.$$

We denote the above integral by $\int X_i^e$. Let $\int X^e = \prod_{i=1}^m \int X_i^e$, $\tilde{X}_i^e = \prod_{j \neq i} \int X_j^e$ and denote the points of $\int \tilde{X}_i^e$ by \tilde{x}_i .

Note that since by assumption (a.3.6) u_i is linear on $\prod_{j=1}^m Y_j$ the domain of the expected utility $g_i(x) = \int_{\omega \in \Omega} u_i(x(\omega)) d\mu(\omega)$ is now $\int X$ and

this latter set will turn out to be equal to $\int X^e$. Since for each i , X_i is compact and convex valued, by the Krein-Milman-Minkowski theorem we have that:

$$(5.1) \quad \text{con } X_i^e(\omega) = X_i(\omega) \text{ } \mu\text{-a.e.}$$

Integrating (5.1) we obtain:

$$(5.2) \quad \int \text{con } X_i^e = \int X_i.$$

Since the measure space (Ω, S_i, μ) is atomless, it follows from Theorem 3 in Aumann (1965) that:

$$(5.3) \quad \int \text{con } X_i^e = \int X_i^e.$$

Combining (5.2) and (5.3) we have that $\int X_i^e = \int X_i$ and therefore $\int X^e = \int X$.

For each i , ($i=1,2,\dots,m$) define the set-valued function

$$F_i : \int \tilde{X}_i^e \rightarrow 2^{\int X_i^e} \text{ by}$$

$$F_i(\tilde{x}_i) = \{y_i \in \int X_i^e : g_i(y_i, \tilde{x}_i) = \max_{x_i \in \int X_i^e} g_i(x_i, \tilde{x}_i)\}.$$

Moreover, define the correspondence $F : \int X^e \rightarrow 2^{\int X^e}$ by

$$F(x) = \prod_{i=1}^m F_i(\tilde{x}_i).$$

We will show that the set-valued function $F : \int X^e \rightarrow 2^{\int X^e}$ satisfies all the conditions of the Kakutani fixed point theorem. It can be easily seen that a fixed point of the set-valued function F is by construction a pure strategy correlated equilibrium for the game Γ . We begin by proving the following claim.

Claim 5.1: $\int X^e$ is compact and nonempty.

Proof: Since for each i , X_i is lower measurable and closed valued, it has a measurable graph [Himmelberg (1975, p. 47)]. Furthermore, since for each i , X_i is integrably bounded it follows from Theorem 4 in Aumann (1965) that $\int X_i$ is compact and so is $\int X^e = \int X$. We now show that $\int X^e$ is nonempty. Since for each i , X_i is S_i -lower measurable and closed valued, by the Kuratowski and Ryll-Nardzewski measurable selection theorem that there exists an S_i -measurable function $f_i : \Omega \rightarrow R^l$

such that $f_i(\omega) \in X_i(\omega)$ μ -a.e. Since X_i is integrably bounded f_i is integrable, i.e., $f_i \in L_1(\mu, \mathbb{R}^k)$ and therefore $f_i \in L_{X_i}$. Hence, L_{X_i} is nonempty and so is $L_X = \prod_{i=1}^m L_{X_i}$. The latter enable us to conclude that $\int X$ is nonempty as well. Consequently, $\int X^e$ is compact and nonempty as was to be shown.

Note that since for each i , u_i is integrably bounded and continuous by virtue of the Lebesgue dominated convergence theorem we can conclude that for each i , g_i is continuous. As in the proof of Theorem 3.1, it can be easily now checked that for each i , F_i is an u.s.c. closed convex valued correspondence and so is F . By the Kakutani fixed point theorem there exists $x^* \in \int X^e$ such that $x^* \in F(x^*)$, which implies that for all i , ($i=1,2,\dots,m$), $g_i(x^*) = \max_{x_i \in \int X_i^e} g_i(x_i, \tilde{x}_i^*)$, i.e., x^* is a pure strategy correlated equilibrium for the game Γ . This completes the proof of Theorem 3.2.

6. CONCLUDING REMARKS

Remark 6.1: Note that Theorem 3.2, i.e., the existence of a pure strategy correlated equilibrium is based heavily on the Lyapunov theorem. In particular, Theorem 3 in Aumann (1965) which enabled us to obtain (5.3), i.e., $\int \text{con } X_i^e = \int X_i^e$, is proved by means of the Lyapunov theorem [see also Schmeidler (1976) for a similar argument with the one used to prove Theorem 3.2].

Remark 6.2: In Theorem 3.2 we assumed that the strategy set of each player is a subset of a finite dimensional Euclidean space. The

reason for this is that we needed Lyapunov's theorem (as noted above), a result which is false in infinite dimensional spaces [see for instance Diestel-Uhl (1977, p. 262)]. The question which arises is whether the argument used to prove Theorem 3.2 remains valid if the strategy set of each player satisfies (a.3.1). The answer is a qualified yes. In particular, note that the key step in the proof of Theorem 3.2 was to show that $\int X^e = \int X$. The latter statement was obtained by using the Krein-Milman-Minkowski Theorem and Theorem 3 of Aumann (1965). However, in an infinite dimensional setting (5.1) in the proof of Theorem 3.2 becomes

$$(5.1') \quad \overline{\text{con}} X_i^e(\omega) = X_i(\omega) \text{ } \mu\text{-a.e.}$$

Therefore,

$$(5.2') \quad \overline{\int \text{con}} X_i^e = \int X_i.$$

By a Theorem of Datko (1973) [see also Khan (1986) for a more general statement], we have that:

$$(5.3') \quad \overline{\int \text{con}} X_i^e = \text{cl} \int X_i^e, \text{ (where cl denotes norm closure).}^7$$

Combining (5.2') and (5.3') we can conclude that the aggregate set of pure strategies is dense in the aggregate set of mixed strategies, i.e., $\text{cl} \int X^e = \int X$. Furthermore, $\text{cl} \int X^e$ is convex [Khan (1986, p. 261)], and by Lemma 3.1 in Yannelis (1988), $\int X$ is weakly compact. Finally it is easy to show that $g_i(\cdot)$ is weakly continuous. In fact, since by assumption (a.3.6) u_i is (norm) continuous and integrably bounded it follows from the Lebesgue dominated converge theorem that g_i is (norm)

continuous. Since u_i is linear so is g_i and by Theorem 15 in Dunford-Schwartz (1958, p. 422) we can conclude that g_i is weakly continuous. The rest of the argument is now similar with that adopted for the proof of Theorem 3.2. However, in this case since $c\ell \int X^e = \int X$ one can only obtain an approximate pure strategy correlated equilibrium for the game Γ .

Remark 6.3: Theorems 3.1 and 3.2 remain true if the utility function of each player u_i is allowed to be state dependent, i.e., u_i is

now defined on $\Omega \times \prod_{j=1}^m Y_j$.

Remark 6.4: Our results can be easily extended to abstract economies as defined in Debreu (1952) and Arrow-Debreu (1954).

FOOTNOTES

¹This notion has been also used by Brandenburger-Dekel (1987, p. 1394) under the name of objective correlated equilibria.

²Note that the convexity of the strategy set of each player amounts to a mixed strategy correlated equilibrium result and this may be objectionable; to quote Aumann (1987, p. 11) "In view of the fact that players may wish to use mixed strategies, the reader may question the assumptions that each player knows his own action. There is no problem if, as usual, one thinks of a mixed strategy simply as a random device for helping the player make up his mind, so that in the end he does know which action he takes." Also, it is important to note that whenever, Nash-type equilibrium existence results are applied to exchange economies the assumption of convexity of the strategy set of each player constitutes no real economic restriction. In particular, exchange economies are converted to abstract economies and the consumption set of each agent in the exchange economy (which is interpreted as the strategy set of each player in the abstract economy) is typically assumed to be convex [see for instance Arrow-Debreu (1954) or Yannelis (1987)].

³It should be noted that recently, Hart-Schmeidler (1987) have obtained correlated equilibrium results different from ours. In particular, they do not consider the case of asymmetric information. Also, it is known that the existence of a correlated equilibrium in a less general setting than ours can be obtained by showing that Nash equilibria are correlated and therefore the classical equilibrium existence result of Nash is directly applicable [see for instance Aumann (1974)]

⁴A correspondence is a set-valued function for which all image sets are nonempty.

⁵By this we mean that for every open subset V of Y the set $\{\omega \in \Omega : X_i(\omega) \cap V \neq \emptyset\}$ is an element of S_i . Note that the fact that X_i is S_i -lower measurable implies that it is also lower measurable.

⁶Let $\{f_n : n=1,2,\dots\}$ be a sequence in $L_1(\mu, Y)$. Then f_n converges weakly to f if and only if $\langle f_n, p \rangle$ converges to $\langle f, p \rangle$ for any $p \in L_\infty(\mu, Y^*)$ (recall that Y^* has the RNP), which is equivalent to the fact that $\langle f_n \chi_A, p \rangle = \langle f_n, \chi_A p \rangle$ converges to $\langle f, \chi_A p \rangle = \langle f \chi_A, p \rangle$ for any $p \in L_\infty(\mu, Y^*)$, $A \in \mathcal{J}$, and each condition above implies that

$\langle f_n \chi_A, x^* \rangle = \langle f_n, \chi_A x^* \rangle$ converges to $\langle f \chi_A, x^* \rangle = \langle f, \chi_A x^* \rangle$ for any $x^* \in Y^*$,
 $A \in \mathcal{J}$.

⁷Rustichini (1987) showed that in the extensions of Theorem 3 of Aumann (1965) given in Datko (1973) and Khan (1986) the norm closure cannot be removed, (i.e., in expression (5.3') the norm closure cannot be dispensed with).

REFERENCES

- Arrow, K. and G. Debreu (1954): "Existence of an Equilibrium for a Competitive Economy," Econometrica 22, 265-290.
- Aumann, R. J. (1974): "Subjectivity and Correlation in Randomized Strategies," Journal of Mathematical Economics 1, 67-96.
- _____ (1987): "Correlated Equilibrium as an Expression of Bayesian Rationality," Econometrica 55, 1-18.
- _____ (1965): "Integrals of Set-Valued Functions," Journal of Mathematical Analysis and Applications 12, 1-12.
- Berge, C. (1963): Topological Spaces, Oliver and Boyd, Edinburgh and London.
- Brandenburger, A. and E. Dekel (1987): "Rationalizability and Correlated Equilibria," Econometrica 55, 1391-1402.
- Datko, R. (1973): "On the Integration of Set-Valued Mappings in a Banach Space," Fundamenta Mathematicae LXXVIII, 205-208.
- Debreu, G. (1952): "A Social Equilibrium Existence Theorem," Proceedings of the National Academy of Sciences of the U.S.A. 38, 121-126.
- Diestel, J. (1977): "Remarks on Weak Compactness in $L_1(\mu, X)$," Glasgow Mathematical Journal 18, 87-91.
- _____ and J. Uhl (1977): Vector Measures, Mathematical Surveys, No. 15, American Mathematical Society, Providence, RI.
- Dunford, N. and J. T. Schwartz (1958): Linear Operators, Part I, Interscience, New York.
- Glicksberg, I. L. (1952): "A Further Generalization of the Kakutani Fixed Point Theorem, With Applications to Nash Equilibrium Points," Proceedings of the American Mathematical Society 3, 170-174.
- Hart, S. and D. Schmeidler (1987): "Existence of Correlated Equilibria," CARESS Working Paper No. 87-14, University of Pennsylvania, April.
- Himmelberg, C. J. (1975): "Measurable Relations," Fundamenta Mathematicae LXXVII, 53-72.
- Khan, M. Ali (1986): "Equilibrium Points of Nonatomic Games over a Banach Space," Transactions of the American Mathematical Society 29, 737-749.

- _____ (1986): "On the Integration of Set-Valued Mappings in a Nonreflexive Banach Space II," Simon Stevin 59, 257-267.
- Nash, J. (1951): "Non-Cooperative Games," Annals of Mathematics 54, 286-295.
- Papageorgiou, N. (1985): "Representation of Set-Valued Operators," Transactions of the American Mathematical Society 292, 557-572.
- Rustichini, A. (1987): "A Counterexample and an Exact Version of Fatou's Lemma in Infinite Dimensions," Archiv der Mathematik, (to appear).
- Schmeidler, D. (1976): "Equilibrium Points of Non-Atomic Games," Journal of Statistical Physics 7, 295-300.
- Yannelis, N. C. (1987): "Equilibria in Non-Cooperative Models of Competition," Journal of Economic Theory 41, 96-111.
- _____ (1988): "Fatou's Lemma in Infinite Dimensional Spaces," Proceedings of the American Mathematical Society 102, 303-310.
- _____ and A. Rustichini (1988): "Equilibrium Points of Non-Cooperative Random and Bayesian Games," Department of Economics, University of Illinois at Urbana-Champaign.

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